

Quantum Phase Transition in the Finite Jaynes-Cummings Lattice Systems

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Phase transitions are commonly held to occur only in the thermodynamical limit of large number of system components. Here we exemplify at the hand of the exactly solvable Jaynes-Cummings (JC) model and its generalization to finite JC-lattices that finite component systems of coupled spins and bosons may exhibit quantum phase transitions (QPT). For the JC-model we find a continuous symmetry-breaking QPT, a photonic condensate with a macroscopic occupation as the ground state and a Goldstone mode as a low-energy excitation. For the two site JC-lattice we show analytically that it undergoes a Mott-insulator to superfluid QPT. We identify as the underlying principle of the emergence of finite size QPT the combination of increasing atomic energy and increasing interaction strength between the atom and the bosonic mode which allows for the exploration of an increasingly large portion of the infinite dimensional Hilbert space of the bosonic mode. This suggests that finite system phase transitions will be present in a broad range of physical systems.

Introduction.— Quantum phase transition (QPT) and spontaneous symmetry breaking are fundamental concepts in physics that lie at the heart of our understanding of various aspects of nature, e.g., phases of matter such as magnetism and superconductivity [1, 2] or the generation of mass [3, 4] in high energy physics. A second-order QPT is characterised by a closing spectral gap and degenerate ground states with a spontaneously broken symmetry. A QPT is typically held to occur only in the thermodynamical limit, i.e. a system with a diverging number of constituent particles or lattice sites [1]. A finite system size generally opens the spectral gap, lifts the ground state degeneracy and restores the symmetry of the ground state [5, 6].

A notable exception is a recent finding in Ref. [7] concerning the Rabi model [8–12], which describes a single-mode cavity field coupled to a two-level atom. While the Dicke model, a N -atom generalization of the Rabi model, has long been known for having a QPT for $N \rightarrow \infty$ [13, 14], Ref. [7] demonstrates that the Rabi model itself undergoes a QPT with the same universal properties when the ratio η of the transition frequency to the cavity frequency diverges [11, 15, 16]. The study [7] further corroborates the duality between the system size N and the frequency ratio η by showing that the finite-size scaling exponents for η are identical to those for N [17, 18]. It is then urgent and important task to see if reaching a limit of the QPT for a system of finite components is a principle that is generally applicable to photonic (phononic) systems with different underlying symmetries, phases, and dimensions. If positively answered, it could open up an important possibility of experimentally investigating the critical phenomena, both in and out of equilibrium, in a small and fully-controlled quantum system.

In this letter, we consider the Jaynes-Cummings (JC) model [19], the Rabi model without the so-called counter-rotating terms, which due to its $U(1)$ symmetry is exactly solvable. We first point out that the well-known analytical solution of the JC model exhibits a ground state instability in the $\eta \rightarrow \infty$ limit, in the sense that the ground state can lower its energy indefinitely by increasing its photon occupation. In this unstable regime, we derive the analytical solution for the ground state and the excitation spectrum by developing a low-

energy effective theory. It shows that the JC model undergoes a second order superradiant QPT in the $\eta \rightarrow \infty$ limit. In the broken-symmetry phase, we find that the ground state forms a photon condensate with a macroscopic photon occupation number and that the excitation spectrum is gapless because the Goldstone mode [20] emerges due to the broken continuous symmetry.

We develop this further by showing that the JC lattice model with only two lattice sites, the JC dimer, undergoes a Mott-insulating-superfluid QPT in the same $\eta \rightarrow \infty$ limit. While the JC lattice model has been known to undergo a Mott-insulating-superfluid QPT in the limit of infinite lattice sites [21–26], here, the QPT in the $\eta \rightarrow \infty$ limit is supported by the infinite dimensional Hilbert space associated with the harmonic oscillator degree of freedom. Our exact analytical solution of the ground state and the excitation spectrum shows that (i) the anti-symmetric normal mode of the coupled-cavities undergoes a transition from a zero excitation insulating phase to a superfluid phase with a broken global $U(1)$ symmetry, while the symmetric mode gets merely squeezed in the superfluid phase and (ii) that the spectral gap of the anti-symmetric mode closes at the critical point, beyond which the excitation is gapless, while the symmetric mode remains gapped for any coupling strength. We emphasize that our analysis is analytic and fully quantum mechanical, going beyond the mean-field approach that is often used in the studies of the JC lattice model for the lack of the exact methods [22, 25].

Quantum phase transition in the JC model.— The Jaynes-Cummings Hamiltonian reads

$$H_{\text{JC}} = \omega_0 a^\dagger a + \frac{\Omega}{2} \sigma_z - \lambda(a\sigma_+ + a^\dagger\sigma_-). \quad (1)$$

Here, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$ with $\sigma_{x,y,z}$ being the Pauli matrices, and a and a^\dagger are the lowering and raising operator of a single mode cavity field, respectively. The cavity frequency is ω_0 , the transition frequency of the two-level atom Ω , and the coupling strength λ . The conserved total number of excitation, $N_{\text{tot}} = a^\dagger a + \sigma_+ \sigma_-$, leads to a $U(1)$ -continuous symmetry, that is, the model is invariant under a gauge transformation $U_\theta = e^{i\theta N_{\text{tot}}}$. Let us denote $|n\rangle$ as a n -photon Fock state and $|\uparrow\rangle$ ($|\downarrow\rangle$) as an eigenstate of σ_z with an eigen-

value $1(-1)$. We also introduce a dimensionless coupling strength $g = \lambda/\sqrt{\omega_0\Omega}$ and a frequency ratio $\eta = \Omega/\omega_0$. Typically, the Jaynes-Cummings model is obtained as an approximation to the Rabi model by neglecting the so-called counter rotating terms, $-\lambda(a\sigma_- + a^\dagger\sigma_+)$ [7]. In systems where the atom-field interaction can be engineered, such as in a circuit QED [27], trapped-ion systems [28] such counter-rotating terms can be strongly suppressed and, remarkably, for an atomic $\Delta m = \pm 1$ transition in interaction with circularly polarized light mode the rotating wave approximation is exact such that Eq. (1) becomes a correct description for any g [29].

The vacuum state $|0, \downarrow\rangle$ is an energy eigenstate of the JC model with an eigenvalue $E_{0,\downarrow} = -\Omega/2$. There are two basis states with n total number of excitation, $|n, \downarrow\rangle$ and $|n-1, \uparrow\rangle$, which span the so-called JC-doublet, denoted as $|n, \pm\rangle$, whose energy eigenvalues in unit of Ω read

$$E_{n,\pm}(\eta, g)/\Omega = (n - \frac{1}{2})\eta^{-1} \pm \frac{1}{2}\sqrt{(1 - \eta^{-1})^2 + 4g^2n\eta^{-1}}. \quad (2)$$

Regardless of η , for $g < 1$ the ground state of Eq. (1) is always $|0, \downarrow\rangle$, until at $g = 1$ there occurs a level crossing between $|0, \downarrow\rangle$ and $|1, -\rangle$. This is followed by a series of level crossings between the lower-energy states of adjacent JC doublets, $|n, -\rangle$ and $|n+1, -\rangle$ [Fig. 1 (a)]. Therefore, increasing the atom-cavity coupling strength increases $\langle N_{\text{tot}} \rangle$ in the ground state, n_G , in discrete steps [Fig. 1 (c)]; in this sense, the JC-type atom-cavity coupling itself assumes the role of a chemical potential. Moreover, as η increases, the increase of n_G becomes progressively sharper near at $g = 1$ [Fig. 1 (c)].

In the $\eta \rightarrow \infty$ limit, the nonlinearity in the spectrum of the JC model disappears, leading to a harmonic spectrum in the low energy sector, i.e., $\lim_{\eta \rightarrow \infty} (E_{n,-}(\eta, g) - E_{0,\downarrow}) = \omega_0(1 - g^2)n$, which is a valid expression for any finite n . The excitation energy for $g < 1$, a normal phase, is therefore $\epsilon_{\text{np}} = \omega_0(1 - g^2)$, which becomes zero at $g = 1$, leading to a degeneracy between $|n, -\rangle$ of any finite n and $|0, \downarrow\rangle$. For $g > 1$, Eq. (2) shows a ground state instability in a sense that the ground state energy can be indefinitely lowered by increasing n , until n becomes infinite, where a new energy minimum can be found. Regarding the Eq. (2) as an effective potential for photon numbers, $V_{\text{eff}}^{\eta,g}(n)$, for a fixed value of η and g , we find the potential minimum at $n = 0$ for $g < 1$ and for $n > 0$ for $g > 1$ and any η [Fig. 1 (b)]. For $\eta \gg 1$, the potential minimum is located at $n_{\text{sp}}(g) \equiv n_G(g > 1) = \eta(g^2 - g^{-2})/4 + \mathcal{O}(\eta^0)$, which explains very well the quadratic behavior of n_G shown in Fig. 1 (c). Furthermore, in the $\eta \rightarrow \infty$ limit and $g > 1$ it is immediate that n_G diverges; that is, a ground state superradiance occurs.

The instability of the JC model for $g > 1$ in the $\eta \rightarrow \infty$ limit predicted from Eq. (2) and the finite value of n_{sp} suggests the derivation of a low-energy effective Hamiltonian for the superradiant phase $g > 1$ that is valid around the potential minimum. To this end, we displace the cavity field a in Eq. (1) by a complex number $\alpha = \alpha_g e^{i\theta}$ with $\alpha_g = \sqrt{n_{\text{sp}}} =$

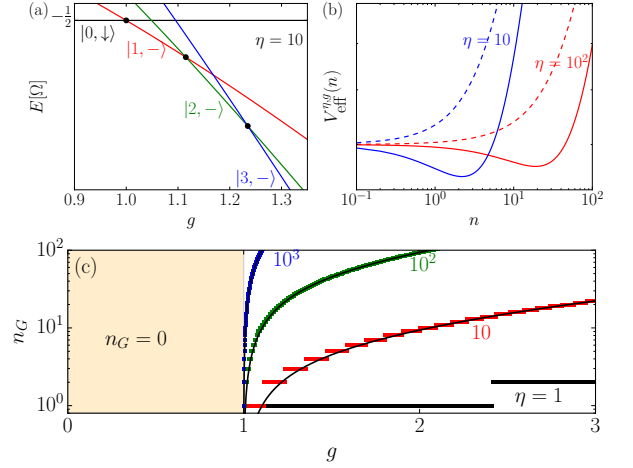


Figure 1. Analytic solution of the JC model. (a) Level crossings for the ground state for a frequency ratio $\eta = 10$. (b) An effective potential $V_{\text{eff}}^{\eta,g}$ for $g = 0.8$ (dashed) and $g = 1.2$ (solid) for different values of $\eta = 10$ and 100 . (c) The total number of excitation of the ground state. As η increases, the change near $g = 1$ becomes progressively sharper, which is well described by $n_{\text{sp}}(g) = \eta(g^2 - g^{-2})/4$ (solid).

$\sqrt{\eta(g^2 - g^{-2})}/4$, i.e., $\bar{H}_{\text{JC}}(\alpha_g, \theta) = \mathcal{D}^\dagger[\alpha] H_{\text{JC}} \mathcal{D}[\alpha]$. By factoring out the phase, we have

$$\begin{aligned} \bar{H}_{\text{JC}}(\alpha_g) &= e^{-i\theta N_{\text{tot}}} \bar{H}_{\text{JC}}(\alpha_g, \theta) e^{i\theta N_{\text{tot}}} \\ &= \omega_0(a^\dagger a + \alpha_g^2) - \frac{\omega_0 \sqrt{\eta}}{2g} (x\tau_x - g^2 p\tau_y) + \frac{g^2 \Omega}{2} \tau_z \\ &\quad + \omega_0 \alpha_g x (\tau_0 + \tau_z) \end{aligned} \quad (3)$$

Here we introduce the new spin operators $\tau_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow| = g^{-2}\sigma_z - \sqrt{1 - g^{-4}}\sigma_x$ as well as $x = a + a^\dagger$ and $p = i(a^\dagger - a)$. Note that $\bar{H}_{\text{JC}}(\alpha_g)$ no longer possesses the $U(1)$ symmetry, and the analytical solution is not available in general. Then, we apply a unitary transformation $U_{\text{JC}} = \exp[\frac{i}{2g\sqrt{\eta}} (g^{-2}x\tau_y + p\tau_x)]$ to Eq. (3) so that a transformed Hamiltonian $U_{\text{JC}}^\dagger \bar{H}_{\text{JC}}(\alpha_g) U_{\text{JC}}$ is free of coupling terms between spin subspaces \mathcal{H}_\downarrow and \mathcal{H}_\uparrow . Finally, a projection onto \mathcal{H}_\downarrow , that is, $\langle\downarrow| U^\dagger \bar{H}_{\text{JC}}(\alpha_g) U |\downarrow\rangle$, leads to the low-energy effective Hamiltonian of JC model in the superradiant phase,

$$\bar{H}_{\text{JC}}^{\text{sp}} = \frac{\omega_0}{4} (1 - g^{-4}) x^2 + E_G^{\text{sp}}(g), \quad (4)$$

Here the ground state energy $E_G^{\text{sp}}(g) = -\Omega(g^2 + g^{-2})/4$, leading to a discontinuity in the second derivative of E_G at $g = 1$, locating a second order QPT.

Interestingly, the effective Hamiltonian is quadratic only in x quadrature of the cavity field, while p quadrature does not appear in the Hamiltonian in the $\eta \rightarrow \infty$ limit [30]. The ground state of $\bar{H}_{\text{JC}}^{\text{sp}}$ is an eigenstate of x quadrature, which is an infinitely squeezed vacuum, whose major axis is the p quadrature, i.e., $|r \rightarrow \infty\rangle = \lim_{r \rightarrow \infty} \mathcal{S}[r] |0\rangle$ with $\mathcal{S}[r] = \exp[-\frac{r}{2}(a^{\dagger 2} - a^2)]$. Going back to the original basis, the

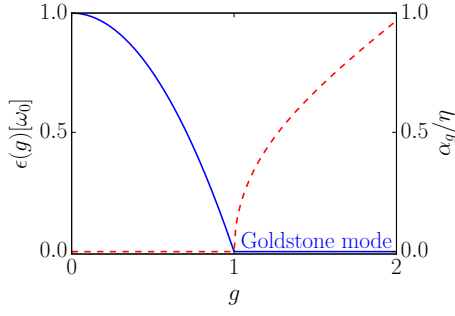


Figure 2. QPT of the JC model. The excitation energy $\epsilon(g)$ (left, blue-solid) and the ground state coherence α_g/η of the cavity field (right, red-dashed) in the $\eta \rightarrow \infty$ limit. For $g > 1$, the $U(1)$ symmetry of the JC model is broken, leading to a Goldstone mode and a non-zero coherence.

ground state of JC model is $|\Psi_G^{\text{sp}}(\theta)\rangle = e^{i\theta a^\dagger} \mathcal{D}[\alpha_g] \mathcal{S}[r \rightarrow \infty] |0\rangle$ for $\theta \in [0, 2\pi]$. Since any choices of the phase θ of the displacement α lead to an identical spectrum, the ground states are also infinitely degenerate. The ground state of the JC Hamiltonian for the superradiant phase is therefore an infinitely squeezed photon condensate, whose renormalized photon occupation number is $n_G/\eta = (g^2 - g_c^2)/4$. Moreover, the $U(1)$ symmetry is spontaneously broken, as it is evident from a non-zero spontaneous coherence $\langle a \rangle / \eta \equiv \langle \Psi_G^{\text{sp}}(\theta) | a | \Psi_G^{\text{sp}}(\theta) \rangle / \eta = e^{i\theta} \sqrt{(g^2 - g_c^2)/4}$, which is an order parameter of the QPT in the JC model [Fig. 2].

We note that the critical behaviors described here, diverging ground state energy, squeezing and spontaneous coherence, arise only in the limit of $\eta \rightarrow \infty$ as the QPT. For any *finite* η , the ground state of the JC model has a finite energy with a definite number of $\langle N \rangle_{\text{tot}}$ for any value of g ; moreover, by the symmetry, the coherence of the ground state $\langle a \rangle$ and the squeezing of the ground state is always zero. We remind that this is exactly analogous with the fact that a model that undergoes a QPT in the $N \rightarrow \infty$ limit restores analytical behaviors for any finite values of N [1, 5, 6].

Because the Eq. (4) is quadratic in only one quadrature without the conjugate variable appearing in the Hamiltonian, the excitation spectrum is gapless [Fig. 2]; that is, it takes infinitesimally small energy to excite the system from the ground state. This gapless excitation is a well-known consequence of spontaneous symmetry breaking of continuous $U(1)$ symmetry and is often called as a Goldstone mode [20]. We note that the effective photon number potential shown in Fig. 1 (b) or the mean-field energy of the JC model [31] assumes the form of the mexican-hat potential in a phase space of the cavity field a ; therefore, the appearance of the Goldstone mode can be intuitively understood from the fact that the excitation along the circle of the potential minima does not cost any energy. Finally, the vanishing spectral gap near the critical point gives rise to a critical exponent, $\epsilon(g) \propto |g - 1|^\alpha$ with $\alpha = 1$, which differs from $\alpha = \frac{1}{2}$ of the Rabi model [7].

We have shown so far that the JC model, one of the most fundamental in quantum optics, exhibits a second-order QPT.

Our analysis clearly demonstrates that the atom-cavity coupling controls the number of photons in the ground state, and that a large η leads to a divergence in the photon number of the ground state. We note that η plays precisely the same role in the Jaynes-Cummings model as the number of atoms in the Tavis-Cummings model [32], a N -atom generalization of the JC model, which undergoes the same kind of QPT [27]. Therefore, the fact that arbitrarily many photons can be created through interaction with other quantum system, regardless of its size, is the origin of the QPTs in a photonic (phononic) system with finite components is possible, in contrast to systems with a hard-core bosons or spins which require infinitely many components to achieve a QPT.

Mott-insulator to Superfluid transition in a finite JC lattice model.— We now consider a photonic lattice model with a finite lattice size. We demonstrate that this model is capable of exhibiting Mott-superfluid type phase transitions away from the conventional thermodynamic limit of infinite lattice sites. Specifically, we consider the JC lattice model [21–24] which describes a one-dimensional lattice of coupled cavities each containing a two-level atom to realize the JC model, which reads $H_{\text{JCL}} = \sum_{i=1}^N H_{\text{JC},i} + \sum_{i=1}^{N-1} J(a_i a_{i+1}^\dagger + h.c.)$, where i indicates i -th cavity and $H_{\text{JC},i} = \omega_0 a_i^\dagger a_i + \frac{\Omega}{2} \sigma_{i,z} - \lambda(a_i \sigma_{i,+} + a_i^\dagger \sigma_{i,-})$. The model has a global $U(1)$ symmetry due to the conservation of the total excitation number, $N_{\text{tot}} = \sum_i (a_i^\dagger a_i + \sigma_{i,+} \sigma_{i,-})$. In the $N \rightarrow \infty$ limit, it is in general not amenable to exact solutions, neither analytically nor numerically; therefore its phase diagram, showing the Mott-insulating-superfluid transition, is often studied based on the mean-field solution [22, 25]. For finite N , the numerically exact calculation shows a crossover from a Mott insulating phase to a superfluid-like phase, due to the finite-size effect which generally prevents the system undergoing a true QPT [23].

We now choose $N = 2$, thus called as JC dimer, which is the smallest possible number of sites for a lattice system, and show that it undergoes a second-order Mott-insulating-superfluid QPT, in the $\eta \rightarrow \infty$ limit. Note that, unlike some of the previous works [22, 25], we introduce neither a chemical potential term to fix the number of polaritons nor counter-rotating terms, which has been shown to stabilize the chemical potential in Ref. [33]; rather, as witnessed in the previous section, a strong JC-type interaction between the field and the atom itself modulates the number of polaritons of each cavity. The JC dimer Hamiltonian can be written in terms of two normal modes, $b_1 = (a_1 - a_2)/\sqrt{2}$ and $b_2 = (a_1 + a_2)/\sqrt{2}$, that is,

$$H_{\text{JCD}} = (\omega_0 - J)b_1^\dagger b_1 + (\omega_0 + J)b_2^\dagger b_2 + \frac{\Omega}{2} \sum_{i=1}^2 \sigma_{i,z} - \frac{\lambda}{\sqrt{2}} (b_1(\sigma_{1,+} - \sigma_{2,+}) + b_2(\sigma_{1,+} + \sigma_{2,+}) + h.c.), \quad (5)$$

where we assume $J/\omega_0 < 1$. In the following we treat the two cases $g < g_c$ and $g > g_c$ which lead to different phases separately. To treat the $g < g_c$ case, we first apply a unitary transformation to H_{JCD} , which decouples the normal modes from

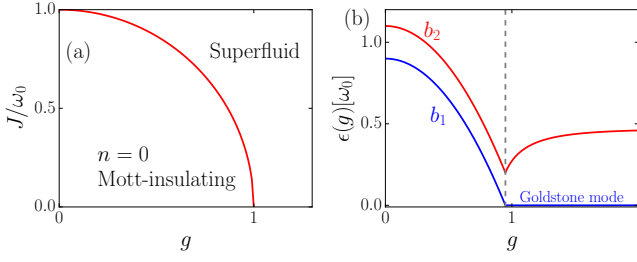


Figure 3. JC lattice model. (a) Phase diagram in the (g, J) plane. (b) Excitation energy of the anti-symmetric (b_1) and symmetric (b_2) normal mode as a function of g for $J/\omega_0 = 0.1$. At the critical point, where the b_1 mode becomes the Goldstone mode, the first derivative of excitation energy of the b_2 mode becomes discontinuous.

the atom, $U_{\text{JCD}} = \exp[\frac{g}{\sqrt{2}\eta}(b_1(\sigma_{1+} - \sigma_{2+}) + b_2(\sigma_{1+} + \sigma_{2+}) - h.c.)]$, and then project the transformed Hamiltonian onto the subspace of the ground state of two atoms $|\downarrow\rangle_1 |\downarrow\rangle_2$ [31]. We note that a similar method has been used in Ref. [34] to study the JC lattice in the dispersive regime. The resulting Hamiltonian is

$$H_{\text{JCD}}^{\text{Mott}} = \omega_0(1 - g^2 - \frac{J}{\omega_0})b_1^\dagger b_1 + \omega_0(1 - g^2 + \frac{J}{\omega_0})b_2^\dagger b_2 - \Omega + \mathcal{O}(\eta^{-\frac{1}{2}}), \quad (6)$$

which becomes exact in the $\eta \rightarrow \infty$ limit. Note that there is a phase boundary $g_c(J) = \sqrt{1 - J/\omega_0}$ [Fig. 3 (a)], on which the spectral gap of the b_1 mode vanishes as $\epsilon \propto (g - g_c(J))^\mu$ with $\mu = 1$ and beyond which the b_1 mode becomes unstable. As a consequence, Eq. (6) is the valid effective Hamiltonian only for $g < g_c(J)$. In this phase, the ground state is the simple vacuum $|0, \downarrow\rangle$ in the original cavity field basis, $|0, \downarrow\rangle_1 |0, \downarrow\rangle_2$. This corresponds to a $n = 0$ Mott-insulating phase, where each cavity assumes a fixed same number of excitation. The b_2 mode remains to be stable for $g < g_c(J)$.

As we have encountered already in the study of the JC model, the fact that the b_1 -mode becomes unstable for $g > g_c(J)$ suggests that it gets occupied by a macroscopic number of photons. Therefore, it is insightful to look at the mean-field energy of H_{JCD} , which we find as $E_{\text{JCD}}^{\text{MF}}(\eta, g, J/\omega_0, \beta)/\Omega = g_c^2(J)\eta^{-1}|\beta|^2 - \sqrt{1 + 2g^2\eta^{-1}|\beta|^2}$ [31]. It is evident that $E_{\text{JCD}}^{\text{MF}}$ assumes the form of the mexican-hat potential for $g > g_c(J)$ where the potential minimum occurs at $\beta_1 = e^{\theta_1}|\beta_1|$ with $|\beta_1| = \sqrt{\eta/(2g_c^2(J))}\sqrt{(g/g_c(J))^2 - (g/g_c(J))^{-2}}$. The mean-field solution predicts a spontaneously broken-symmetry phase and an appearance of the Goldstone mode. It is also easy to show that the second derivative of the ground state energy in g become discontinuous at $g = g_c(J)$, indicating that it is a second order QPT [31].

For $g > g_c(J)$, we obtain a low-energy effective Hamiltonian of the JC dimer using a similar strategy successfully used for the JC model in the first part of this letter. That is, we first displace the b_1 mode by its mean-field amplitude β_1 , which leads to a new quantization axis for two atoms and a

new atomic state for the ground state [31]. Then, just like we did in the Mott phase, we find a unitary transformation decoupling the normal modes and atoms, followed by a projection onto the low-energy subspace [31]. The resulting effective Hamiltonian reads

$$\tilde{H}_{\text{JCD}}^{\text{SF}} = \frac{\omega_0 g_c^2}{4}(1 - \frac{g_c^4}{g^4})x_1^2 + \frac{J}{2}p_2^2 + \frac{\omega_0}{4}\left(1 + \frac{J}{\omega_0} - \frac{g_c^6}{g^4}\right)x_2^2 \quad (7)$$

up to the constant ground state energy and g_c here denotes $g_c(J)$.

The two normal modes are decoupled from each other, and the above Hamiltonian is exactly solvable. First, the p_1 quadrature of the b_1 mode disappears from the effective Hamiltonian, as in the case of the JC model in the superradiant phase shown in Eq. (4). Therefore, it immediately follows that the global $U(1)$ symmetry of the JC lattice model is broken for $g > g_c(J)$. The non-zero coherence of each cavity field $\langle a_i \rangle \neq 0$ marks the onset of the *superfluid* phase and becomes an order parameter. The excitation spectrum of the b_1 mode is gapless, showing that the Goldstone mode correctly emerges as the low-energy excitation in the broken symmetry phase [Fig. 3 (b)]. The Hamiltonian for the b_2 mode in Eq. (7) can be easily diagonalized to give a harmonic spectrum with an excitation frequency of $\epsilon_2^{\text{SF}}(g) = J\sqrt{2(1 + \omega_0/J(1 - g_c^6(J)/g^4))}$. As shown in Fig. 3 (b), the b_2 mode remains gapped for both phases. Interestingly, the first derivative of $\epsilon_2^{\text{SF}}(g)$ is discontinuous at $g = g_c(J)$. Such a slope discontinuity of the b_2 mode can be potentially used to detect the presence of the Goldstone mode as suggested in Ref. [27]. The ground state of the b_2 mode is a squeezed vacuum, whose squeezing parameter is given by $\xi = -1/4 \ln(\frac{1}{2}(1 + \omega_0/J(1 - g_c^6(J)/g^4)))$, which is zero at $g = g_c$ and gradually increases. The JC-dimer may also serve as testing ground for the physics of phase interfaces in lattice systems [35].

Conclusion—Unlike massive particles, photons can be created by its interaction with an atom, as the chemical potential of the photon vanishes [36]. We have shown that for an atom with a much larger characteristic frequency than the photon but strongly coupled to it, it is possible to have a macroscopic photon occupation in the ground state. This, as we have demonstrated at the hand of the JC models, leads to the emergence of a QPT in a system composed of finitely many components, photonic modes and atoms. We expect that our finding here, together with one presented in the Ref. [7], opens up an important possibility to study critical phenomena of light and sound, such as QPT, universality, and dynamics of the QPT, in a fully controlled, small quantum systems including a superconducting circuits and trapped-ions.

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